

# Pressure dependence of superconductivity in doped two-leg ladder cuprates

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## Abstract

Within the kinetic energy driven superconducting mechanism, the effect of the pressure on superconductivity in the doped two-leg ladder cuprates is studied. It is shown that the superconducting transition temperature in the doped two-leg ladder cuprate superconductors increases with increasing pressure in the underpressure regime, and reaches a maximum in the optimal pressure, then decreases in the overpressure regime. This domed shape of the pressure dependence of the superconducting transition temperature is similar to that of the pressure dependence of the longitudinal part of the superconducting gap parameter, indicating that the pressure dependence of superconductivity in the doped two-leg ladder cuprates is mainly produced by the development of the pairing correlation along legs.

*Key words:* Pressure dependence of superconductivity, Two-leg ladder cuprates, Kinetic energy driven superconducting mechanism

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After over ten years of intense experimental study of the doped two-leg ladder cuprates, a significant body of reliable and reproducible data has been accumulated by using many probes, which indicate that the doped two-leg ladder cuprates show many nonconventional physical properties [1,2,3,4,5]. When charge carriers are added to the two-leg ladder cuprates, a metal-insulator transition is observed [6,7,8,9,10,11,12]. Although the ambient pressure ladder superconductivity was not observed until now, superconductivity in one of the doped two-leg ladder cuprate  $\text{Sr}_{14-x}\text{Ca}_x\text{Cu}_{24}\text{O}_{41}$  has been observed under pressure [6,7,8,9]. In particular, the maximal superconducting (SC) transition temperature in  $\text{Sr}_{14-x}\text{Ca}_x\text{Cu}_{24}\text{O}_{41}$  occurs around the optimal pressure, and then decreases in both underpressure and overpressure regimes [6,7,8,9]. Moreover, the structure of the doped two-leg ladder cuprates

under high pressure remains the same as the case in ambient pressure [8], and the spin background in the SC phase does not drastically alter its spin gap properties [2], which show that the dominant physics arises from the individual ladders, similar to the way that the dominant physics is determined by the two-dimensional  $\text{CuO}_2$  planes in the doped planar cuprate superconductors [13]. In this case, a challenging issue for theory is to explain the pressure dependence of superconductivity in the doped two-leg ladder cuprate  $\text{Sr}_{14-x}\text{Ca}_x\text{Cu}_{24}\text{O}_{41}$ .

Experimentally, it has been shown [2,4,5] that at ambient pressure, the exchange coupling  $J_{\parallel}$  along the legs is greater than exchange coupling  $J_{\perp}$  across a rung, i.e.,  $J_{\parallel} > J_{\perp}$ , and similarly the hopping  $t_{\parallel}$  along the legs is greater than the rung hopping strength  $t_{\perp}$ , i.e.,  $t_{\parallel} > t_{\perp}$ . In this case, the two-leg ladder cuprates are highly anisotropic materials.

Furthermore, the experimental results have showed that the most important role of pressure for realizing superconductivity in the doped two-leg ladder cuprates is to reduce the distance between ladders and chains, and then the coupling between ladders and chains is enhanced [6,7,8,9,10,11,12]. This leads to that the values of  $J_{\perp}/J_{\parallel}$  and  $t_{\perp}/t_{\parallel}$  increase with increasing pressure. In other words, the pressurization induces anisotropy shrinkage on the two-leg ladder cuprates, and then there is a tendency toward the isotropy for two-leg ladders [6,7,8,9,10,11,12]. These experimental results explicitly imply that the values of  $J_{\perp}/J_{\parallel}$  and  $t_{\perp}/t_{\parallel}$  of the doped two-leg ladder cuprates are closely related to the pressurization, and therefore the pressure effects may be imitated by the variation of the values of  $J_{\perp}/J_{\parallel}$  and  $t_{\perp}/t_{\parallel}$ .

Theoretically, it has been shown within the  $t$ - $J$  ladder model that the charge carrier pair correlation is very robust [2,14], clearly indicative of a ground-state dominated by strong SC tendencies. In particular, it has been found based on the density matrix renormalization group (DMRG) and Monte Carlo simulations [15] that the pairing correlations are enhanced when the top of the bonding quasiparticle band and the bottom of the antibonding band are near the Fermi level. Furthermore, using the renormalized mean-field theory, many authors have shown that superconductivity should exist in the d-wave channel [16], which has been confirmed by variety of numerical simulations [17]. However, to the best of our knowledge, no systematic calculations have been performed within the  $t$ - $J$  ladder model for the pressure dependence of the SC transition temperature to confront the experimental data.

In this Letter, we study the pressure dependence of superconductivity in the doped two-leg ladder cuprates within the *anisotropic*  $t$ - $J$  ladder model. Our results show that the SC transition temperature in the doped two-leg ladder cuprate superconductors increases with increasing pressure in the underpressure regime, and reaches a maximum in the optimal pressure, then decreases in the overpressure regime, in qualitative agreement with the experimental results [8]. Moreover, this domed shape of the pressure dependence of the SC transition temperature is similar to that of the pressure dependence of the longitudinal part of the SC gap parameter, and therefore it is also shown that superconductivity in the doped two-leg ladder cuprates is mainly produced by the development of the pairing correlation along legs. The strong electron correlation in the  $t$ - $J$  ladder model manifests itself by

the electron single occupancy local constraint [1,2], this is why the crucial requirement is to impose this electron local constraint for a proper understanding of the physical properties of the doped two-leg ladder cuprates. To incorporate this local constraint, we have developed a charge-spin separation (CSS) fermion-spin theory [18], where the constrained electron operators are decoupled as the gauge invariant spinful fermion operator and spin operator, with the spinful fermion operator keeps track of the charge degree of freedom together with some effects of the spin configuration rearrangements due to the presence of the doped hole itself (dressed holon), while the spin operator keeps track of the spin degree of freedom. The advantage of this approach is that the electron local constraint for the single occupancy is satisfied in analytical calculations [18]. Within this CSS fermion-spin theoretical framework, a kinetic energy driven SC mechanism [19] has been proposed, where the dressed holon-spin interaction from the kinetic energy term induces the dressed holon pairing state by exchanging spin excitations, then the electron Cooper pairs originating from the dressed holon pairing state are due to the charge-spin recombination, and their condensation reveals the SC ground-state. Based on this kinetic energy driven SC mechanism, we [20] have discussed the possible doping dependence of superconductivity in the doped two-leg ladder cuprates within the *isotropic*  $t$ - $J$  ladder model, and the result shows that the spin-liquid ground-state at the half-filling evolves into the SC ground-state upon doping. Since a detailed description of the method within the  $t$ - $J$  ladder model has been given in Ref. [20], and therefore we do not repeat here.

In Fig. 1, we plot the longitudinal (solid line) and transverse (dashed line) parts of the SC gap parameters as a function of  $t_{\perp}/t_{\parallel}$  at the doping concentration  $\delta = 0.25$  in parameter  $t_{\parallel}/J_{\parallel} = 2.5$  with temperature  $T = 0.0001J_{\parallel}$ . For the convenience in the discussions, we have chosen the variation of  $(t_{\perp}/t_{\parallel})^2$  under the pressure is the same as that of  $J_{\perp}/J_{\parallel}$ , i.e.,  $(t_{\perp}/t_{\parallel})^2 = J_{\perp}/J_{\parallel}$ . Our results show that both longitudinal and transverse parts of the SC gap parameter have a similar pressure dependent behavior. In particular, the value of the longitudinal part of the SC gap parameter  $\Delta_L$  increases with increasing  $t_{\perp}/t_{\parallel}$  in the lower  $t_{\perp}/t_{\parallel}$  regime, and reaches a maximum in the optimal  $(t_{\perp}/t_{\parallel})_{\text{opt}} \approx 0.7$ , then decreases in the higher  $t_{\perp}/t_{\parallel}$  regime. In this case, the underpressure, optimal pressure, and overpressure regimes are corresponding to the lower  $t_{\perp}/t_{\parallel}$ , op-

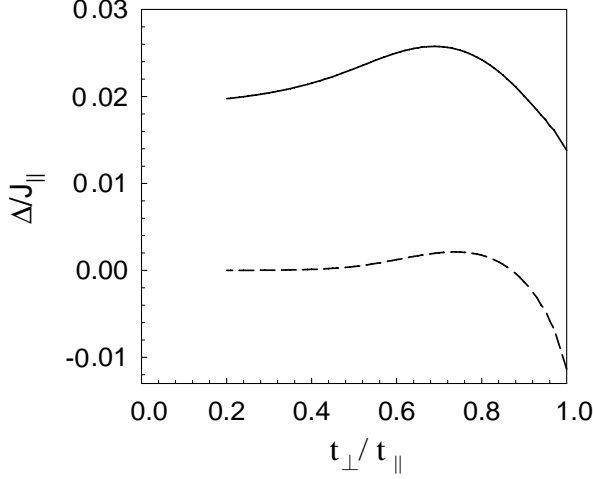


Fig. 1. The longitudinal (solid line) and transverse (dashed line) SC gap parameters as a function of  $t_{\perp}/t_{\parallel}$  at  $\delta = 0.25$  in  $t_{\parallel}/J_{\parallel} = 2.5$  with  $T = 0.0001J_{\parallel}$ .

timal ( $t_{\perp}/t_{\parallel}$ )<sub>opt</sub>, and higher  $t_{\perp}/t_{\parallel}$  regimes, respectively.

In correspondence with the SC gap parameter, the SC transition temperature  $T_c$  as a function of  $t_{\perp}/t_{\parallel}$  at  $\delta = 0.25$  in  $t_{\parallel}/J_{\parallel} = 2.5$  is plotted in Fig. 2. For comparison, the experimental result [8] of the SC transition temperature  $T_c$  in the doped two-leg ladder cuprate superconductor  $\text{Sr}_{14-x}\text{Ca}_x\text{Cu}_{24}\text{O}_{41}$  with  $x = 13.6$  (the corresponding doping concentration  $\delta \sim 0.25$ ) as a function of pressure is also shown in Fig. 2 (inset). Our result shows that the SC transition temperature in the doped two-leg ladder cuprate superconductors increases with increasing  $t_{\perp}/t_{\parallel}$  in the lower  $t_{\perp}/t_{\parallel}$  regime, and reaches a maximum in the optimal ( $t_{\perp}/t_{\parallel}$ )<sub>opt</sub>  $\approx 0.7$ , then decreases in the higher  $t_{\perp}/t_{\parallel}$  regime. Using a reasonably estimative value of  $J_{\parallel} \sim 90\text{meV} \approx 1000\text{K}$  in the doped two-leg ladder cuprate superconductors [3], the SC transition temperature in the optimal pressure is  $T_c \approx 0.036J_{\parallel} \approx 36\text{K}$ , in qualitative agreement with the experimental data [8]. Moreover, in comparison with the results of the longitudinal and transverse SC gap parameters in Fig. 1, we therefore find that this domed shape of the pressure dependence of the SC transition temperature is similar to that of the pressure dependence of the longitudinal part of the SC gap parameter. As the pressure increases, although the hole movement along the rung direction is enhanced and superconductivity appears in the ladder, our result also shows that superconductivity in the doped two-leg ladder cuprates is mainly produced by the development of the pairing corre-

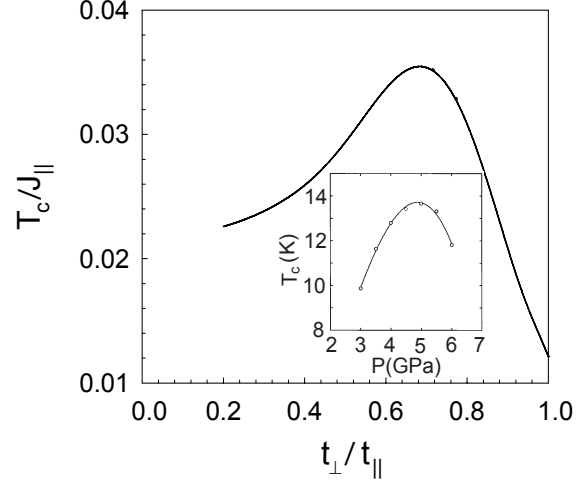


Fig. 2. The SC transition temperature as a function of  $t_{\perp}/t_{\parallel}$  at  $\delta = 0.25$  in  $t_{\parallel}/J_{\parallel} = 2.5$ . Inset: the experimental result of  $\text{Sr}_{14-x}\text{Ca}_x\text{Cu}_{24}\text{O}_{41}$  with  $x = 13.6$  (the corresponding doping concentration  $\delta \sim 0.25$ ) taken from Ref. [8].

lation along legs, and is consistent with the one-dimensional charge dynamics under high pressure [8,2].

For a better understanding of superconductivity in the doped two-leg ladder cuprates, we have made a series of calculations for the doping dependence of the SC transition temperature at different  $t_{\perp}/t_{\parallel}$ , and the result of the SC transition temperature  $T_c$  as a function of the hole doping concentration  $\delta$  for  $t_{\parallel}/J_{\parallel} = 2.5$  and ( $t_{\perp}/t_{\parallel}$ )<sub>opt</sub>  $\approx 0.7$  is plotted in Fig. 3. It is shown that the maximal SC transition temperature  $T_c$  occurs around the optimal doping concentration  $\delta_{\text{opt}} \approx 0.12$ , and then decreases in both underdoped and overdoped regimes. Moreover,  $T_c$  in the underdoped regime is proportional to the hole doping concentration  $\delta$ , and therefore  $T_c$  in the underdoped regime is set by the hole doping concentration as in the doped planar cuprates [19], which reflects that the density of the dressed holons directly determines the superfluid density in the underdoped regime. In comparison with the result of the doped *isotropic* two-leg ladder case [20], our present result also shows that the range of superconductivity in the doped *anisotropic*  $t$ - $J$  ladder cuprates is larger than that of the doped *isotropic*  $t$ - $J$  ladder case, in particular, the optimal doping in the present *anisotropic* case moves to higher doping regime than that of the doped *isotropic*  $t$ - $J$  ladder cuprates. Furthermore, this domed shape of the doping dependence of the SC transition temperature in Fig. 3 is similar to that of the pressure dependence

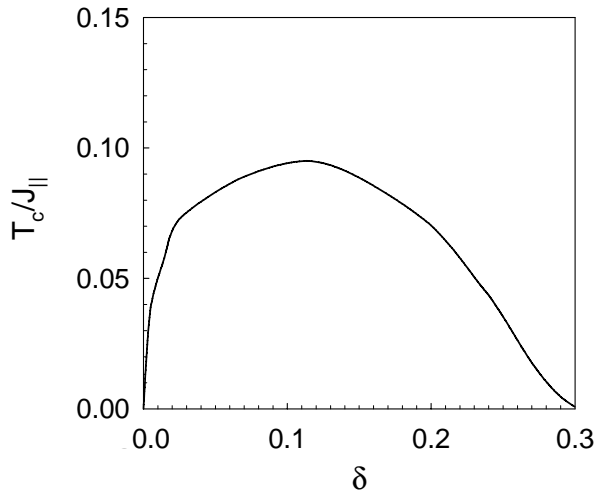


Fig. 3. The SC transition temperature as a function of the doping concentration for  $t_{\parallel}/J_{\parallel} = 2.5$  and  $(t_{\perp}/t_{\parallel})_{\text{opt}} \approx 0.7$ .

of the SC transition temperature in Fig. 2, reflecting a corresponding relationship between the pressure effect and hole doping concentration. This is consistent with the experiments [6,7,8,9,10,11,12], since it has been shown from the experiments that the main effect of pressure in the doped two-leg ladder cuprates is to reduce the distance between the ladders and chains, which leads to the doped hole redistribution between chains and ladders [6,7,8,9], in particular, when Ca is doped upon the original Sr-based Ca-undoped phase, the interatomic distance ladder-chain was found to be reduced by Ca substitution, leading to a redistribution of holes originally present only on the chains [10,11].

As we [20] have shown the SC-order in the doped two-leg ladder cuprates is established through an emerging SC quasiparticle, and therefore the SC-order is controlled by both gap function and quasiparticle coherence. In this case, the essential physics of the pressure dependence of the SC transition temperature in the doped two-leg ladder cuprates can be understood from a competition between the kinetic energy and magnetic energy [20]. As we have mentioned above, the experimental results [6,7,8,9] have indicated that the main effect of the pressure in the doped two-leg ladder cuprate superconductors is to reduce the distance between the ladders and chains (then enhance the coupling between the ladders and chains), which leads to the doped hole redistribution between chains and ladders. On the other hand, when Ca is doped upon the original Sr-based Ca-undoped phase, the interatomic distance ladder-chain was found to be reduced (then the cou-

pling between the ladders and chains is enhanced) by Ca substitution, leading to a redistribution of holes originally present only on the chains [10,11,12]. These experimental results show that an increase of the pressure (then an increase of the coupling between the ladders and chains) may be corresponding to an increase in the number of charge carriers on the ladders [8,9,10,11,12]. In this case, the kinetic energy increases with increasing pressure (doping), but at the same time, the spin correlation is destroyed, therefore the pressure effect (doping) on the doped two-leg ladder cuprates can be considered as a competition between the kinetic energy and magnetic energy, and the magnetic energy decreases with increasing pressure (doping). In the underpressure (underdoping) and optimal pressure (optimal doping) regimes, the charge carrier concentration is small, and therefore magnetic energy is rather large, then the dressed holon (then electron) attractive interaction by exchanging spin excitations is also rather strong to form the dressed holon pairs (then electron Cooper pairs) for the most dressed holons (then electrons), therefore the SC transition temperature increases with increasing pressure (doping). However, in the overpressure (overdoping) regime, the charge carrier concentration is large and magnetic energy is relatively small, then the dressed holon (then electron) attractive interaction by exchanging spin excitations is also relatively weak, in this case, not all dressed holons (then electrons) can be bounden as dressed holon pairs (then electron Cooper pairs) by this weak attractive interaction, and therefore the SC transition temperature decreases with increasing pressure (doping).

Based on the DMRG and Monte Carlo numerical simulation [15], the domed shape of the pressure dependence of the SC pairing correlation in the doped two-leg ladder cuprates has been discussed within the Hubbard ladder model. They [15] calculate the ground state expectation value of the rung-rung pair-field correlation function  $D(i, j) = \langle \Delta(i) \Delta^{\dagger}(j) \rangle$ , and show that near the half-filling, the strength of the pairing correlation depends sensitively upon  $t_{\perp}/t_{\parallel}$  and the doping as well as  $U/t_{\parallel}$  (then  $t_{\parallel}/J_{\parallel}$ ), where there are strong interchain antiferromagnetic correlations and a large single-particle spectral weight at the Fermi points of the bonding and antibonding bands, which is consistent with our present result. However, their result also shows [15] that the optimal value of the SC pairing correlation occurs for intermediate values of  $U/t_{\parallel}$  and for doping near half-filling with  $t_{\perp}/t_{\parallel} = 1.5$ ,

which is inconsistent with our present result. The reason for this inconsistency is not clear, and the related issue is under investigation now.

In summary, we have discussed the effect of the pressure on superconductivity in the doped two-leg ladder cuprates within the kinetic energy driven SC mechanism. Our results show that the SC transition temperature in the doped two-leg ladder cuprates increases with increasing pressure in the underpressure regime, and reaches a maximum in the optimal pressure, then decreases in the overpressure regime. This domed shape of the pressure dependence of the SC transition temperature is similar to that of the pressure dependence of the longitudinal part of the SC gap parameter, and therefore it is also shown that the pressure dependence of superconductivity in the doped two-leg ladder cuprates is mainly produced by the development of the pairing correlation along legs.

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## References

- [1] E. Dagotto and T.M. Rice, *Science* **271**, 618 (1996), and referenes therein.
- [2] E. Dagotto, *Rep. Prog. Phys.* **62**, 1525 (1999), and referenes therein.
- [3] S.Katano, T. Nagata, J. Akimitsu, M. Nishi, and K. Kakurai, *Phys. Rev. Lett.* **82**, 636 (1999); M. Matsuda, K. Katsumata, H. Eisaki, N. Motoyama, S. Uchida, S.M. Shapiro, and G. Shirane, *Phys. Rev. B* **54**, 12199 (1996).
- [4] K. Magishi, S. Matsumoto, Y. Kitaoka, K. Ishida, K. Asayama, M. Uehara, T. Nagata, and J. Akimitsu, *Phys. Rev. B* **57**, 11533 (1998); S. Ohsugi, K. Magishi, S. Matsumoto, Y. Kitaoka, T. Nagata, and J. Akimitsu, *Phys. Rev. Lett.* **82**, 4715 (1998).
- [5] R.S. Eccleston, M. Uehara, J. Akimitsu, H. Eisaki, N. Motoyama, and S. Uchida, *Phys. Rev. Lett.* **81**, 1702 (1998).
- [6] M. Uehara, T. Nagata, J. Akimitsu, H. Takahashi, N. Mori, and K. Kinoshita, *J. Phys. Soc. Jpn.* **65**, 2764 (1996).
- [7] T. Nagata, M. Uehara, J. Goto, J. Akimitsu, N. Motoyama, H. Eisaki, S. Uchida, H. Takahashi, T. Nakanishi, and N. Mori, *Phys. Rev. Lett.* **81**, 1090 (1998).
- [8] M. Isobe, T. Ohta, M. Onoda, F. Izumi, S. Nakano, J.Q. Li, Y. Matsui, E. Takayama-Muromachi, T. Matsumoto, and H. Hayakawa, *Phys. Rev. B* **57**, 613 (1998).
- [9] Y. Piskunov, D. Jérôme, P. Auban-Senzier, P. Wzietek, and A. Yakubovsky, *Phys. Rev. B* **72**, 064512 (2005).
- [10] T. Ohta, M. Onoda, F. Izumi, M. Isobe, E. Takayama-Muromachi, and A.W. Hewat, *J. Phys. Soc. Jpn.* **66**, 3107 (1997).
- [11] M. Kato, K. Shiot, and Y. Koike, *Physica C* **258**, 284 (1996); N. Motoyama, T. Osafune, T. Kakeshita, H. Eisaki, and S. Uchida, *Phys. Rev. B* **55**, R3386 (1997); T. Osafune, N. Motoyama, H. Eisaki, and S. Uchida, *Phys. Rev. Lett.* **78**, 1980 (1997).
- [12] T. Nagata, M. Uehara, J. Goto, N. Komiya, J. Akimitsu, N. Motoyama, H. Eisaki, S. Uchida, H. Takahashi, T. Nakanishi and N. Möri, *Physica C* **282-287**, 153 (1997).
- [13] See, e.g., M.A. Kastner, R.J. Birgeneau, G. Shiran, and Y. Endoh, *Rev. Mod. Phys.* **70**, 897 (1998).
- [14] E. Dagotto, J. Riera, and D.J. Scalapino, *Phys. Rev. B* **45**, 5744 (1992).
- [15] R.M. Noack, N. Bulut, D.J. Scalapino, and M.G. Zacher, *Phys. Rev. B* **56**, 7162 (1997).
- [16] M. Sigrist, T.M. Rice, and F.C. Zhang, *Phys. Rev. B* **49**, 12058 (1994).
- [17] J.A. Riera, *Phys. Rev. B* **49**, 3629 (1994); H. Tsunetsugu, M. Troyer, and T.M. Rice, *Phys. Rev. B* **51**, 16456 (1995); C.A. Hayward, D. Poilblanc, R.M. Noack, D.J. Scalapino, and W. Hanke, *Phys. Rev. Lett.* **75**, 926 (1995).
- [18] Shiping Feng, Jihong Qin, and Tianxing Ma, *J. Phys. Condens. Matter* **16**, 343 (2004); Shiping Feng, Tianxing Ma, and Jihong Qin, *Mod. Phys. Lett. B* **17**, 361 (2003).
- [19] Shiping Feng, *Phys. Rev. B* **68**, 184501 (2003); Shiping Feng, Tianxing Ma, and Huaiming Guo, *Physica C* **436**, 14 (2006); Shiping Feng and Tianxing Ma, *Phys. Lett. A* **350**, 138 (2006).
- [20] Jihong Qin, Feng Yuan, and Shiping Feng, *Phys. Lett. A* **358**, 448 (2006).